CHAPTER 6

Chapter Review

VOCABULARY

- scientific notation, p. 325
- polynomial function, p. 329
- leading coefficient, p. 329
- constant term, p. 329
- degree of a polynomial function, p. 329
- standard form of a polynomial function, p. 329
- synthetic substitution, p. 330
- end behavior, p. 331
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6.1 USING PROPERTIES OF EXPONENTS

Examples on pp. 323–325

EXAMPLE You can use properties of exponents to evaluate numerical expressions and to simplify algebraic expressions.

$$\frac{(3x^2y)^5}{9x^{10}y^6} = \frac{3^5x^{2\cdot 5}y^5}{9x^{10}y^6} = \frac{243}{9}x^{10-10}y^{5-6} = 27x^0y^{-1} = \frac{27}{y}$$ all positive exponents

Simplify the expression. Tell which properties of exponents you used.

1. $\left(\frac{2}{3}\right)^2 \cdot (6xy^{-1})^3$ **2.** $x^4(x^{-5}x^3)^2$ **3.** $\frac{-63xy^9}{18x^{-2}y^3}$ **4.** $\frac{5x^2}{y^{-2}} \cdot \frac{1}{25x^2y}$

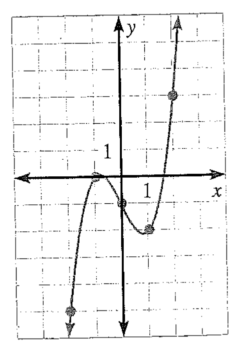
6.2 EVALUATING AND GRAPHING POLYNOMIAL FUNCTIONS

Examples on pp. 329–332

EXAMPLES Use direct or synthetic substitution to evaluate a polynomial function.

Evaluate $f(x) = x^3 - 2x - 1$ when $x = 3$ (synthetic substitution):

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -2 & -1 \\ & & 3 & 9 & 21 \\ \hline & 1 & 3 & 7 & 20 \end{array} \leftarrow f(3) = 20$$



To graph, make a table of values, plot points, and identify end behavior.

x	−3	−2	−1	0	1	2	3
$f(x)$	−22	−5	0	−1	−2	3	20

The leading coefficient is positive and the degree is odd, so $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

Use synthetic substitution to evaluate the polynomial function for the given value of *x*.

5. $f(x) = x^3 + 3x^2 - 12x + 7, x = 3$ **6.** $f(x) = x^4 - 5x^3 - 3x^2 + x - 5, x = -1$

Graph the polynomial function.

7. $f(x) = -x^3 + 2$ **8.** $f(x) = x^4 - 3$ **9.** $f(x) = x^3 - 4x + 1$

6.3 ADDING, SUBTRACTING, AND MULTIPLYING POLYNOMIALS

Examples on pp. 338–340

EXAMPLES You can add, subtract, or multiply polynomials.

$$\begin{array}{rrrr} 4x^3 & + 2x^2 & & + 1 \\ - & (x^2 & + x & - 5) \\ \hline 4x^3 & + x^2 & - x & + 6 \end{array}$$

$$\begin{aligned}(x - 3)(x^2 + 5x - 1) &= (x - 3)(x^2) + (x - 3)(5x) + (x - 3)(-1) \\ &= x^3 - 3x^2 + 5x^2 - 15x - x + 3 \\ &= x^3 + 2x^2 - 16x + 3\end{aligned}$$

Perform the indicated operation.

10. $(3x^3 + x^2 + 1) - (x^3 + 3)$ **11.** $(x - 3)(x^2 + x - 7)$ **12.** $(x + 3)(x - 5)(2x + 1)$

6.4 FACTORING AND SOLVING POLYNOMIAL EQUATIONS

Examples on pp. 345–347

EXAMPLES You can solve some polynomial equations by factoring.

Factor $8x^3 - 125$.

$$\begin{aligned}8x^3 - 125 &= (2x)^3 - 5^3 \\ &= (2x - 5)\big((2x)^2 + (2x \cdot 5) + 5^2\big) \\ &= (2x - 5)(4x^2 + 10x + 25)\end{aligned}$$

Solve $x^3 - 3x^2 - 5x + 15 = 0$.

$$\begin{aligned}x^2(x - 3) - 5(x - 3) &= 0 \\ (x - 3)(x^2 - 5) &= 0 \\ x = 3 \text{ or } x &= \pm\sqrt{5}\end{aligned}$$

Find the real-number solutions of the equation.

13. $x^3 + 64 = 0$ **14.** $x^4 - 6x^2 = 27$ **15.** $x^3 + 3x^2 - x - 3 = 0$

6.5 THE REMAINDER AND FACTOR THEOREMS

Examples on pp. 352–355

EXAMPLES You can use polynomial long division, and in some cases synthetic division, to divide polynomials.

$$\begin{array}{r} x^2 - 7x + 6 \\ x + 9\overline{\big)\, x^3 + 2x^2 - 57x + 54} \\ \underline{x^3 + 9x^2} \qquad\qquad \\ -7x^2 - 57x \qquad \\ \underline{-7x^2 - 63x} \qquad \\ 6x + 54 \\ \underline{6x + 54} \\ 0 \end{array}$$

$$\frac{x^3 + 2x^2 - 57x + 54}{x + 9} = x^2 - 7x + 6$$

Divide $3x^3 + 2x^2 - x + 4$ by $x + 5$.

$$\begin{array}{r|rrrr} -5 & 3 & 2 & -1 & 4 \\ & & -15 & 65 & -320 \\ \hline & 3 & -13 & 64 & -316 \end{array}$$

$$\frac{3x^3 + 2x^2 - x + 4}{x + 5} = 3x^2 - 13x + 64 + \frac{-316}{x + 5}$$

Divide. Use synthetic division if possible.

16. $(x^4 + 5x^3 - x^2 - 3x - 1) \div (x - 1)$ **17.** $(2x^3 - 5x^2 + 5x + 4) \div (2x - 5)$

6.6–6.7 FINDING ZEROS OF POLYNOMIAL FUNCTIONS

Examples on pp. 359–361 and pp. 366–368

EXAMPLE You can use the rational zero theorem and the fundamental theorem of algebra to find all the zeros of a polynomial function.

$f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$ Possible rational zeros: $\frac{\pm 1, \pm 2, \pm 11, \pm 22}{1}$

Using synthetic division, you can find that the rational zeros are 1 and 2.
The degree of f is 4, so f has 4 zeros. To find the other two zeros, write in factored form: $f(x) = (x - 1)(x - 2)(x^2 + 6x + 11)$. Solve $x^2 + 6x + 11 = 0$: $x = -3 \pm \sqrt{2}i$.
So the zeros of $f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$ are $1, 2, -3 + \sqrt{2}i, -3 - \sqrt{2}i$.

Find all the real zeros of the function.

18. $f(x) = x^3 + 12x^2 + 21x + 10$

19. $f(x) = x^4 + x^3 - x^2 + x - 2$

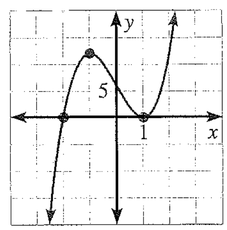
6.8 ANALYZING GRAPHS OF POLYNOMIAL FUNCTIONS

Examples on pp. 373–375

EXAMPLE You can identify x-intercepts and turning points when you analyze the graph of a polynomial function.

The graph of $f(x) = 3x^3 - 9x + 6$ has

- two x-intercepts, -2 and 1.
- a local maximum at $(-1, 12)$.
- a local minimum at $(1, 0)$.



Graph the polynomial function. Identify the x-intercepts and the points where the local maximums and local minimums occur.

20. $f(x) = (x - 2)^2(x + 2)$

21. $f(x) = x^3 - 3x^2$

22. $f(x) = 3x^4 + 4x^3$

6.9 MODELING WITH POLYNOMIALS

Examples on pp. 380–382

EXAMPLE Sometimes you can use finite differences or cubic regression to find a polynomial model for a set of data.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	
−1	2	7	14	23	34	function values
3	5	7	9	11		first-order differences
2	2	2	2			second-order differences

Since second-order differences are nonzero and constant, the data set can be modeled by a polynomial function of degree 2. The function is $f(x) = x^2 - 2$.

23. Show that the third-order differences for the function $f(n) = n^3 + 1$ are nonzero and constant.

24. Write a cubic function whose graph passes through points $(1, 0)$, $(-1, 0)$, $(4, 0)$, and $(2, -12)$. Use cubic regression on a graphing calculator to verify your answer.