CHAPTER 5

Chapter Review

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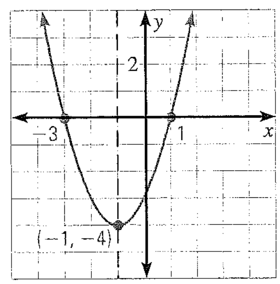
5.1 GRAPHING QUADRATIC FUNCTIONS

Examples on pp. 249–252

EXAMPLE You can graph a quadratic function given in standard form, vertex form, or intercept form. For instance, the same function is given below in each of these forms, and its graph is shown.



Standard form: $y = x^2 + 2x - 3$;

axis of symmetry: $x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$

Vertex form: $y = (x + 1)^2 - 4$; vertex: $(-1, -4)$

Intercept form: $y = (x + 3)(x - 1)$; x-intercepts: $-3, 1$

Graph the quadratic function.

1. $y = x^2 + 4x + 7$

2. $y = -3(x - 2)^2 + 5$

3. $y = \frac{1}{2}(x + 1)(x - 5)$

5.2–5.3 SOLVING BY FACTORING AND BY FINDING SQUARE ROOTS

Examples on pp. 256–259, 264–266

EXAMPLES You can use factoring or square roots to solve quadratic equations.

Solving by factoring:

$x^2 - 4x - 21 = 0$

$(x + 3)(x - 7) = 0$

$x + 3 = 0$ or $x - 7 = 0$

$x = -3$ or $x = 7$

Solving by finding square roots:

$4x^2 - 7 = 65$

$4x^2 = 72$

$x^2 = 18$

$x = \pm\sqrt{18} = \pm 3\sqrt{2}$

Solve the quadratic equation.

4. $x^2 + 11x + 24 = 0$ **5.** $x^2 - 8x + 16 = 0$ **6.** $2x^2 + 3x + 1 = 0$

7. $3u^2 = -4u + 15$ **8.** $25v^2 - 30v = -9$ **9.** $2x^2 = 200$

10. $5x^2 - 2 = 13$ **11.** $4(t + 6)^2 = 160$ **12.** $-(k - 1)^2 + 7 = -43$

5.4 COMPLEX NUMBERS

Examples on pp. 272–276

EXAMPLES You can add, subtract, multiply, and divide complex numbers. You can also find the absolute value of a complex number.

Addition: $(1 + 8i) + (2 - 3i) = (1 + 2) + (8 - 3)i = 3 + 5i$

Subtraction: $(1 + 8i) - (2 - 3i) = (1 - 2) + (8 + 3)i = -1 + 11i$

Multiplication: $(1 + 8i)(2 - 3i) = 2 - 3i + 16i - 24i^2 = 2 + 13i - 24(-1) = 26 + 13i$

Division: $\frac{1 + 8i}{2 - 3i} = \frac{1 + 8i}{2 - 3i} \cdot \frac{2 + 3i}{2 + 3i} = \frac{-22 + 19i}{13} = -\frac{22}{13} + \frac{19}{13}i$

Absolute value: $|1 + 8i| = \sqrt{1^2 + 8^2} = \sqrt{65}$

In Exercises 13–16, write the expression as a complex number in standard form.

13. $(7 - 4i) + (-2 + 5i)$ **14.** $(2 + 11i) - (6 - i)$

15. $(3 + 10i)(4 - 9i)$ **16.** $\frac{8 + i}{1 - 2i}$

17. Find the absolute value of $6 + 9i$.

5.5 COMPLETING THE SQUARE

Examples on pp. 282–285

EXAMPLES You can use completing the square to solve quadratic equations and change quadratic functions from standard form to vertex form.

Solving an equation:

$$x^2 + 6x + 13 = 0$$
$$x^2 + 6x = -13$$
$$x^2 + 6x + 9 = -13 + 9$$
$$(x + 3)^2 = -4$$
$$x + 3 = \pm\sqrt{-4}$$
$$x = -3 \pm 2i$$

Writing a function in vertex form:

$$y = x^2 + 6x + 13$$
$$y + \underline{?} = (x^2 + 6x + \underline{?}) + 13$$
$$y + 9 = (x^2 + 6x + 9) + 13$$
$$y + 9 = (x + 3)^2 + 13$$
$$y = (x + 3)^2 + 4$$

Note that the vertex is $(-3, 4)$.

Solve the quadratic equation by completing the square.

18. $x^2 + 4x = 3$ **19.** $x^2 - 10x + 26 = 0$ **20.** $2w^2 + w - 7 = 0$

Write the quadratic function in vertex form and identify the vertex.

21. $y = x^2 - 8x + 17$ **22.** $y = -x^2 - 2x - 6$ **23.** $y = 4x^2 + 16x + 23$

5.6 THE QUADRATIC FORMULA AND THE DISCRIMINANT

Examples on pp. 291–294

EXAMPLE You can use the quadratic formula to solve any quadratic equation.

$$3x^2 - 5x = -1$$

$$3x^2 - 5x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)} = \frac{5 \pm \sqrt{13}}{6}$$

Use the quadratic formula to solve the equation.

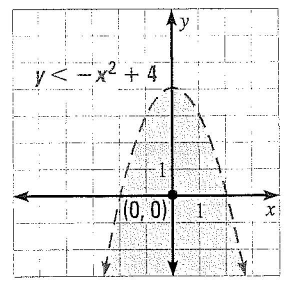
24. $x^2 - 8x + 5 = 0$ **25.** $9x^2 = 1 - 7x$ **26.** $5v^2 + 6v + 7 = v^2 - 4v$

5.7 GRAPHING AND SOLVING QUADRATIC INEQUALITIES

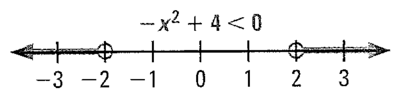
Examples on pp. 299–302

EXAMPLES You can graph a quadratic inequality in two variables and solve a quadratic inequality in one variable.

Graphing an inequality in two variables: To graph $y < -x^2 + 4$, draw the dashed parabola $y = -x^2 + 4$. Test a point inside the parabola, such as (0, 0). Since (0, 0) is a solution of the inequality, shade the region inside the parabola.



Solving an inequality in one variable: To solve $-x^2 + 4 < 0$, graph $y = -x^2 + 4$ and identify the x-values where the graph lies below the x-axis. Or, solve $-x^2 + 4 = 0$ to find the critical x-values -2 and 2, then test an x-value in each interval determined by -2 and 2 to find the solution. The solution is $x < -2$ or $x > 2$.



Graph the quadratic inequality.

27. $y \geq x^2 - 4x + 4$ **28.** $y < x^2 + 6x + 5$ **29.** $y > -2x^2 + 3$

Solve the quadratic inequality.

30. $x^2 - 3x - 4 \leq 0$ **31.** $2x^2 + 7x + 2 \geq 0$ **32.** $9x^2 > 49$

5.8 MODELING WITH QUADRATIC FUNCTIONS

Examples on pp. 306–308

EXAMPLE You can write a quadratic function given characteristics of its graph.

To find a function for the parabola with vertex $(1, -3)$ and passing through $(0, -1)$, use the vertex form $y = a(x - h)^2 + k$ with $(h, k) = (1, -3)$ to write $y = a(x - 1)^2 - 3$. Use the point $(0, -1)$ to find a: $-1 = a(0 - 1)^2 - 3$, so $-1 = a - 3$, and therefore $a = 2$. The function is $y = 2(x - 1)^2 - 3$.

Write a quadratic function whose graph has the given characteristics.

33. vertex: (6, 1)
point on graph: (4, 5)

34. x-intercepts: -4, 3
point on graph: (1, 20)

35. points on graph:
$(-5, 1)$, $(-4, -2)$, $(3, 5)$