2.3: Product and Quotient Rules and Higher Order Derivatives

<u>Objective:</u> Find derivatives using product/quotient rules
Find derivatives of trig functions
Find higher order derivatives

* You know how to find the derivative of $x(x^2+4)$, but what about $(x^3+4x)(3x^2+2x-5)$

Product Rule

The product of two diff functions f and g is the derivative of the first function times the second function plus the first fuction times the derivative of the second function

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

<u>Ex</u>

$h(x) = (3x-2x^2)(5+4x)$

$h'(x) = (3x-2x^2)'(5+4x) + (3x-2x^2)(5+4x)'$

$(3-4x)(5+4x) + (3x-2x^2)(4)$

$15+12x-20x-16x^2+12x-8x^2$

$\boxed{-24x^2+4y+15}$

<u>Ex 2</u>

$y = 2x\cos x - 2\sin x$

$y' = (2x)'(\cos x) + (2x)(\cos x)' - (2\sin x)'$

$2\cos x - 2x\sin x - 2\cos x$

$\boxed{-2x\sin x}$

Now Try

1) $3x^2 \sin x$

$$\boxed{\begin{array}{c} 6x\sin x + 3x^2\cos x \\ 3x(2\sin x + x\cos x) \end{array}}$$

2) $(x^3+4x)(3x^2+2x-5)$

$(3x^2+4)(3x^2+2x-5)+(x^3+4x)(6x+2)$

$9x^4+6x^3-15x^2+12x^2+8x-20+6x^4+2x^3+24x^2+8x$

$$\boxed{15x^4+8x^3+21x^2+16x-20}$$

3) $(s^3-2)^2$

Quotient Rule

The derivative of $\frac{f}{g}$ is the derivative of the numerator times the denominator minus the numerator times the derivative of the denominator all divided by the square of the denominator

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{[g(x)]^2}$$

Ex $\dfrac{5x-2}{x^2+1} = \dfrac{(5x-2)'(x^2+1)-(5x-2)(x^2+1)'}{(x^2+1)^2}$

$$\frac{5(x^2+1)-[(5x-2)(2x)]}{(x^2+1)^2} = \frac{5x^2+5-[10x^2-4x]}{(x^2+1)^2}$$

$$\boxed{\frac{-5x^2+4x+5}{(x^2+1)^2}}$$

Ex 2

$$\frac{x^2}{2+x} \Rightarrow \frac{(x^2)'(2+x)-(x^2)(2+x)'}{(2+x)^2}$$

$$\frac{2x(2+x)-[(x^2)(1)]}{(2+x)^2} = \frac{2x+2x^2-x^2}{(2+x)^2}$$

$$\boxed{\frac{x^2+2x}{(2+x)^2}}$$

Now Try

1) $\frac{4x-2}{x^2+1} = \boxed{\frac{-4x^2+4x+4}{(x^2+1)^2}}$ 2) $\frac{x^4}{\sin x} = \boxed{\frac{4x^3\sin x - x^4\cos x}{\sin^2 x}}$

* Always see if you can simplify the function before deriving

Ex $\frac{-3(3x-2x^2)}{7x} = \frac{-3x(3-2x)}{7x}$

$$-\frac{3}{7}(3-2x) \rightarrow -\frac{3}{7}(-2) = \boxed{\frac{6}{7}}$$

Derivatives of Trig Functions

$\frac{d}{dx}[\tan x] = \sec^2 x$ $\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\sec x] = \sec(x)\tan(x)$ $\frac{d}{dx}[\csc x] = -\csc(x)\cot(x)$

Now Try

$$\frac{1-\cos x}{\sin x} = \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \csc x - \cot x$$

$$\boxed{-\csc(x)\cot(x) + \csc^2 x} = \boxed{\csc(x)\left[\csc(x) - \cot(x)\right]}$$

Higher Order Derivatives

* We found velocity by differentiating the position function. We can find acceleration by diff. the equation for velocity,
  - Or diff. the position function twice

$s(t)$: position
$s'(t) = v(t)$: velocity
$s''(t) = v'(t) = a(t)$: acceleration

Closure: What is the product rule?
quotient rule?
What is the derivative of tan?
csc?

HW: pg 126 #'s 1-53 odd
#'s 7-18
39-44