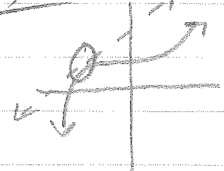
2.1: The Derivative and the Tangent Line Problem

Objective: • Find the slope of the tangent line to a curve @ a point
• Understand the difference between differentiability and continuity

• What is a line tangent to a curve?
- It is a line that touches a curve in exactly one spot at that given spot

Ex



• To find the equation of a line tangent to a curve, you must find the derivative of the equation

Notation of Derivative

$f'(x)$ / $\frac{dy}{dx}$ / y' / $\frac{d}{dx}[f(x)]$ / $D_x[y]$

Theorem 2.1: Diff. Implies Continuity
If f is diff. @ $x=c$, then f is continuous @ c

Lets start finding derivatives of functions

Constant Rule
: The derivative of a constant function is zero. i.e.

$$\frac{d}{dx}[c]=0$$

Ex

$y = 7 \rightarrow y' = 0$

$f(x) = 0 \rightarrow f'(x) = 0$

$s(t) = k\pi^2$ (k is constant) $\rightarrow s'(t) = 0$

What if our function isn't constant?

Power Rule

If n is a rational #, then the function $f(x) = x^n$ is differentiable and

$$\frac{d}{dx}[x^n] = n x^{n-1}$$

Ex

$x^3 \rightarrow 3x^{3-1} \rightarrow 3x^2$

$\sqrt[3]{x} \rightarrow x^{1/3} \rightarrow \frac{1}{3}x^{-2/3} \rightarrow \frac{1}{3(\sqrt[3]{x})^2}$

$\frac{1}{x^2} \rightarrow x^{-2} \rightarrow -2x^{-3} \rightarrow \frac{-2}{x^3}$

$x \rightarrow x^0 \rightarrow 1$

Now try

1) $f(x) = -9$

0

2) $y = \frac{1}{x^8}$

$\frac{-8}{x^9}$

3) $\frac{1}{\sqrt[4]{x}}$

$\frac{1}{4x^{3/4}}$

Finding the Slope of a Graph

The slope of the graph @ a point is the value of the derivative @ that point

Ex

Find the slope of x^4 when $x = -1$

$f(x) = x^4$

$f'(x) = 4x^3 \xrightarrow{@x=-1} f'(-1) = 4(-1)^3$

$= -4$

Now Try

Find the slope of the tangent line @ the given point

1) $y = \frac{8}{x^2}$ @ $x = 2$

$y' = \frac{16}{x} \rightarrow 8$

2) $y = \sqrt{x}$ @ $x = 4$

$y' = \frac{1}{2\sqrt{x}} = \frac{1}{4}$

Finding an Equation of a Tangent Line

Ex Find the equation of the tangent line to the graph of $f(x) = x^2$ when $x = -2$

* First, find the y-value @ the given x-value

$f(x) = x^2$

$f(-2) = -4 \rightarrow$ so the tangent line will pass through $(-2, 4)$

* Next, find the slope

$f(x) = x^2$

$f'(x) = 2x$

$f'(-2) = -4 \rightarrow m = -4$

* You have the slope and a coordinate

Point-Slope

$y - 4 = -4(x+2)$

Slope-Intercept

$4 = -4(-2) + b$

$4 = 8 + b$

$b = -4$

$y = -4x - 4$

Constant Rule

If f is a diff. function and c is a real #, then cf is also diff and

$$\frac{d}{dx}[cf(x)] = cf'(x)$$

Ex

$$y = \frac{2}{x} \rightarrow 2x^{-1} \rightarrow 2\frac{d}{dx}x^{-1} \rightarrow \frac{-2}{x^2}$$

$$f(x) = \frac{4t^2}{5} \rightarrow \frac{4}{5}\frac{d}{dx}t^2 \rightarrow \frac{4}{5}(2t) \rightarrow \frac{8t}{5}$$

$$y = -\frac{3}{2}x \rightarrow -\frac{3}{2}\frac{d}{dx}x \rightarrow -\frac{3}{2}(1) \rightarrow -\frac{3}{2}$$

Now Try

1) $y = \frac{5}{2x^3}$

$\boxed{y' = \frac{15}{2x^4}}$

2) $y = 4x^3$

$\boxed{y' = 12x}$

3) $\frac{7}{(3x)^{-2}}$

$\boxed{126x}$

Sum and Difference Rule

The sum or difference of two diff functions "f" and "g" is itself diff.

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x) \rightarrow \text{Sum Rule}$$

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x) \rightarrow \text{Difference Rule}$$

Ex

$f(x) = x^3 - 4x + 5 \rightarrow f'(x) = 3x^2 - 4$

$y = -\frac{x^4}{2} + 3x^3 - 2x \rightarrow y' = -\frac{4x^3}{2} + 9x^2 - 2$

$-2x^3 + 9x^2 - 2$

Now Try

1) $-2t^2 + 3t - 6$

$\boxed{-4t+3}$

2) $2x^3 - x^2 + 3x$

$\boxed{6x^2 - 2x + 3}$

3) $\frac{x^3 - 3x^2 + 4}{x^2}$

$\boxed{1 - \frac{8}{x^3}}$

4) $x(x^2+1)$

$\boxed{3x^2+1}$

Derivative of Sin and Cos

$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$

Ex

$y = 2\sin x$

$y' = 2\cos x$

$y = \frac{\sin x}{2}$

$y' = \frac{1}{2}\cos x$

$y = x + \cos x$

$y' = 1 - \sin x$

$-15y^{-4}$

$\frac{5}{8x^3} \quad \frac{5x^{-3}}{8}$

Now Try

1) $\frac{\pi}{2}\sin x - \cos x$

$\boxed{\frac{\pi}{2}\cos x + \sin x}$

2) $7 + \sin x$

$\boxed{-\cos x}$

3) $\frac{5}{(2x)^3} + 2\cos x$

$\boxed{\frac{-15}{8x^4} - 2\sin x}$

Rates of Change

* A common use for rate of change is used to describe motion of an object.
* Upward movement is described with positive velocity
* Downward movement is described with negative velocity

right

left

$$\text{Rate} = \frac{\text{distance}}{\text{time}}$$

$$\text{Average Velocity} = \frac{\text{change in distance}}{\text{change in time}}$$

Ex

Billiard Ball is dropped and represented with the function

$s = -16t^2 + 100$; s is height in feet
h is time in seconds

Find the average velocity over the following intervals.

[1, 2] [1, 1.5] [1, 1.1]

$s(1) = 84$

$s(2) = 36$

$$\frac{\Delta s}{\Delta t} = \frac{84 - 36}{1 - 2} = \frac{48}{-1} = -48$$

dropping @ an average velocity of 48 ft/sec

* We just learned how to find <u>average</u> velocity
* <u>We can find instantaneous velocity by taking the derivative of the position function</u>
* <u>The speed of an object is the absolute value of the velocity</u>

<u>Ex</u>
<u>At time $t=0$, a diver jumps from a platform that is 32 ft above the water. The position is written as</u>

$s(t) = -16t^2 + 16t + 32$; s is height in ft
t is time in seconds

1) When does the diver hit the water

$0 = -16(t^2 - t - 2)$

$0 = (t-2)(t+1)$

$t = 2, -1 \longrightarrow$ hits water in 2 seconds

2) What is the diver's velocity at impact

$s'(t) = -32t + 16$

$s'(2) = -32(2) + 16$

$s'(2) = -64 + 16$

$s'(2) = -48 \longrightarrow$ @ 2 seconds, velocity is 48 ft/second

<u>Closure</u>: • What is the derivative of a constant
• How to we find the slope of a graph @ a given point
• How can you find velocity

<u>HW</u>: pg 115 #'s 3-23, 31-58 odd