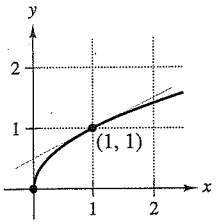
2.2 Exercises

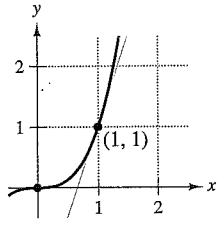
See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1 and 2, use the graph to estimate the slope of the tangent line to $y = x^n$ at the point $(1, 1)$. Verify your answer analytically. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

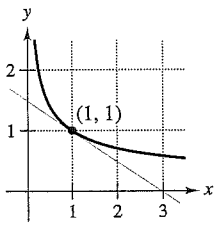
1. (a) $y = x^{1/2}$



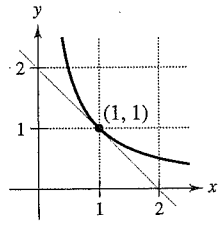
(b) $y = x^3$



2. (a) $y = x^{-1/2}$



(b) $y = x^{-1}$



In Exercises 3–24, use the rules of differentiation to find the derivative of the function.

3. $y = 12$
4. $f(x) = -9$
5. $y = x^7$
6. $y = x^{16}$
7. $y = \dfrac{1}{x^5}$
8. $y = \dfrac{1}{x^8}$
9. $f(x) = \sqrt[5]{x}$
10. $g(x) = \sqrt[4]{x}$
11. $f(x) = x + 11$
12. $g(x) = 3x - 1$
13. $f(t) = -2t^2 + 3t - 6$
14. $y = t^2 + 2t - 3$
15. $g(x) = x^2 + 4x^3$
16. $y = 8 - x^3$
17. $s(t) = t^3 + 5t^2 - 3t + 8$
18. $f(x) = 2x^3 - x^2 + 3x$
19. $y = \dfrac{\pi}{2}\sin\theta - \cos\theta$
20. $g(t) = \pi\cos t$
21. $y = x^2 - \frac{1}{2}\cos x$
22. $y = 7 + \sin x$
23. $y = \dfrac{1}{x} - 3\sin x$
24. $y = \dfrac{5}{(2x)^3} + 2\cos x$

In Exercises 25–30, complete the table.

Original Function	Rewrite	Differentiate	Simplify
25. $y = \dfrac{5}{2x^2}$			
26. $y = \dfrac{2}{3x^2}$			
27. $y = \dfrac{6}{(5x)^3}$			
28. $y = \dfrac{\pi}{(3x)^2}$			
29. $y = \dfrac{\sqrt{x}}{x}$			
30. $y = \dfrac{4}{x^{-3}}$			

In Exercises 31–38, find the slope of the graph of the function at the given point. Use the *derivative* feature of a graphing utility to confirm your results.

Function	Point
31. $f(x) = \dfrac{8}{x^2}$	$(2, 2)$
32. $f(t) = 3 - \dfrac{3}{5t}$	$\left(\frac{3}{5}, 2\right)$
33. $f(x) = -\frac{1}{2} + \frac{7}{5}x^3$	$\left(0, -\frac{1}{2}\right)$
34. $y = 3x^3 - 10$	$(2, 14)$
35. $y = (4x + 1)^2$	$(0, 1)$
36. $f(x) = 3(5 - x)^2$	$(5, 0)$
37. $f(\theta) = 4\sin\theta - \theta$	$(0, 0)$
38. $g(t) = -2\cos t + 5$	$(\pi, 7)$

In Exercises 39–54, find the derivative of the function.

39. $f(x) = x^2 + 5 - 3x^{-2}$
40. $f(x) = x^2 - 3x - 3x^{-2}$
41. $g(t) = t^2 - \dfrac{4}{t^3}$
42. $f(x) = x + \dfrac{1}{x^2}$
43. $f(x) = \dfrac{4x^3 + 3x^2}{x}$
44. $f(x) = \dfrac{x^3 - 6}{x^2}$
45. $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2}$
46. $h(x) = \dfrac{2x^2 - 3x + 1}{x}$
47. $y = x(x^2 + 1)$
48. $y = 3x(6x - 5x^2)$
49. $f(x) = \sqrt{x} - 6\sqrt[3]{x}$
50. $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$
51. $h(s) = s^{4/5} - s^{2/3}$
52. $f(t) = t^{2/3} - t^{1/3} + 4$
53. $f(x) = 6\sqrt{x} + 5\cos x$
54. $f(x) = \dfrac{2}{\sqrt[3]{x}} + 3\cos x$

In Exercises 55–58, (a) find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.

Function	Point
55. $y = x^4 - 3x^2 + 2$	$(1, 0)$
56. $y = x^3 + x$	$(-1, -2)$
57. $f(x) = \dfrac{2}{\sqrt[4]{x^3}}$	$(1, 2)$
58. $y = (x^2 + 2x)(x + 1)$	$(1, 6)$

In Exercises 59–64, determine the point(s) (if any) at which the graph of the function has a horizontal tangent line.

59. $y = x^4 - 2x^2 + 3$

60. $y = x^3 + x$

61. $y = \dfrac{1}{x^2}$

62. $y = x^2 + 9$

63. $y = x + \sin x, \quad 0 \le x < 2\pi$

64. $y = \sqrt{3}x + 2\cos x, \quad 0 \le x < 2\pi$

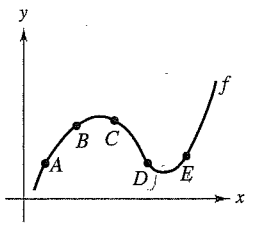
In Exercises 65–70, find k such that the line is tangent to the graph of the function.

Function	*Line*
65. $f(x) = x^2 - kx$	$y = 5x - 4$
66. $f(x) = k - x^2$	$y = -6x + 1$
67. $f(x) = \dfrac{k}{x}$	$y = -\dfrac{3}{4}x + 3$
68. $f(x) = k\sqrt{x}$	$y = x + 4$
69. $f(x) = kx^3$	$y = x + 1$
70. $f(x) = kx^4$	$y = 4x - 1$

71. Sketch the graph of a function f such that $f' > 0$ for all x and the rate of change of the function is decreasing.

CAPSTONE

72. Use the graph of f to answer each question. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com.*



(a) Between which two consecutive points is the average rate of change of the function greatest?

(b) Is the average rate of change of the function between A and B greater than or less than the instantaneous rate of change at B?

(c) Sketch a tangent line to the graph between C and D such that the slope of the tangent line is the same as the average rate of change of the function between C and D.

WRITING ABOUT CONCEPTS

In Exercises 73 and 74, the relationship between f and g is given. Explain the relationship between f' and g'.

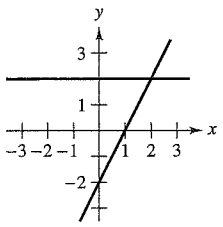
73. $g(x) = f(x) + 6$

74. $g(x) = -5f(x)$

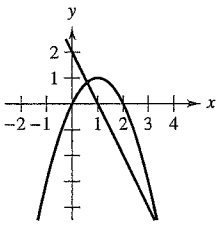
WRITING ABOUT CONCEPTS (continued)

In Exercises 75 and 76, the graphs of a function f and its derivative f' are shown on the same set of coordinate axes. Label the graphs as f or f' and write a short paragraph stating the criteria you used in making your selection. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com.*

75.



76.



77. Sketch the graphs of $y = x^2$ and $y = -x^2 + 6x - 5$, and sketch the two lines that are tangent to both graphs. Find equations of these lines.

78. Show that the graphs of the two equations $y = x$ and $y = 1/x$ have tangent lines that are perpendicular to each other at their point of intersection.

79. Show that the graph of the function

$$f(x) = 3x + \sin x + 2$$

does not have a horizontal tangent line.

80. Show that the graph of the function

$$f(x) = x^5 + 3x^3 + 5x$$

does not have a tangent line with a slope of 3.

In Exercises 81 and 82, find an equation of the tangent line to the graph of the function f through the point (x_0, y_0) not on the graph. To find the point of tangency (x, y) on the graph of f, solve the equation

$$f'(x) = \frac{y_0 - y}{x_0 - x}.$$

81. $f(x) = \sqrt{x}$

$(x_0, y_0) = (-4, 0)$

82. $f(x) = \dfrac{2}{x}$

$(x_0, y_0) = (5, 0)$

83. ***Linear Approximation*** Use a graphing utility, with a square window setting, to zoom in on the graph of

$$f(x) = 4 - \tfrac{1}{2}x^2$$

to approximate $f'(1)$. Use the derivative to find $f'(1)$.

84. ***Linear Approximation*** Use a graphing utility, with a square window setting, to zoom in on the graph of

$$f(x) = 4\sqrt{x} + 1$$

to approximate $f'(4)$. Use the derivative to find $f'(4)$.