

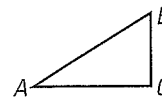
In this lesson, you have learned to prove that two triangles are congruent by the *definition of congruence*—that is, by showing that all pairs of corresponding angles and corresponding sides are congruent. In upcoming lessons, you will learn more efficient ways of proving that triangles are congruent. The properties below will be useful in such proofs.

THEOREM

THEOREM 4.4 Properties of Congruent Triangles

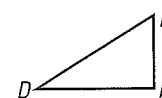
REFLEXIVE PROPERTY OF CONGRUENT TRIANGLES

Every triangle is congruent to itself.



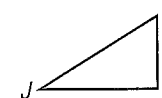
SYMMETRIC PROPERTY OF CONGRUENT TRIANGLES

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.



TRANSITIVE PROPERTY OF CONGRUENT TRIANGLES

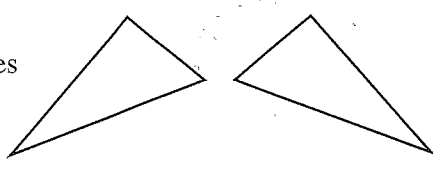
If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



GUIDED PRACTICE

Vocabulary Check ✓

- Copy the congruent triangles shown at the right. Then label the vertices of your triangles so that $\triangle JKL \cong \triangle RST$. Identify all pairs of congruent *corresponding angles* and *corresponding sides*.

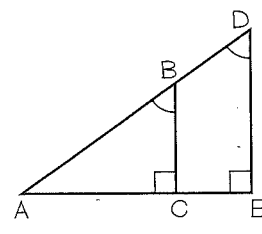


Concept Check ✓

ERROR ANALYSIS Use the information and the diagram below.

On an exam, a student says that $\triangle ABC \cong \triangle ADE$ because the corresponding angles of the triangles are congruent.

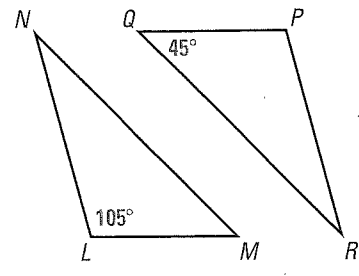
- How does the student know that the corresponding angles are congruent?
- Is $\triangle ABC \cong \triangle ADE$? Explain your answer.



Skill Check ✓

Use the diagram at the right, where $\triangle LMN \cong \triangle PQR$.

- What is the measure of $\angle P$?
- What is the measure of $\angle M$?
- What is the measure of $\angle R$?
- What is the measure of $\angle N$?
- Which side is congruent to \overline{QR} ?
- Which side is congruent to \overline{LN} ?

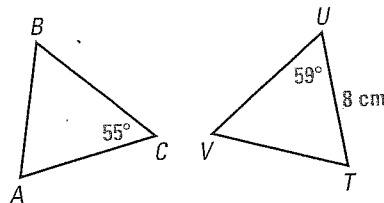


PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 809.

DESCRIBING CONGRUENT TRIANGLES In the diagram, $\triangle ABC \cong \triangle TUV$. Complete the statement.



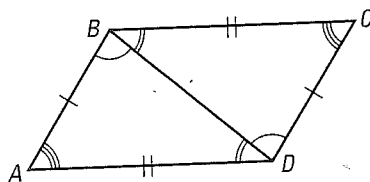
10. $\angle A \cong ?$
11. $\overline{VT} \cong ?$
12. $\triangle VTU \cong ?$
13. $BC = ?$
14. $m\angle A = m\angle ? = ?^\circ$

15. Which of the statements below can be used to describe the congruent triangles in Exercises 10–14? (There may be more than one answer.)

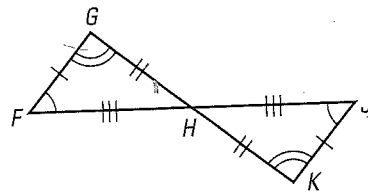
- A. $\triangle CBA \cong \triangle TUV$ B. $\triangle CBA \cong \triangle VUT$
 C. $\triangle UTV \cong \triangle BAC$ D. $\triangle TVU \cong \triangle ACB$

NAMING CONGRUENT FIGURES Identify any figures that can be proved congruent. Explain your reasoning. For those that can be proved congruent, write a congruence statement.

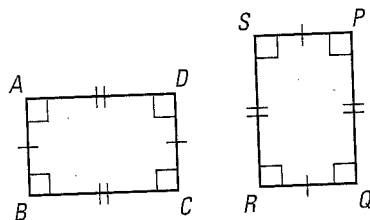
16.



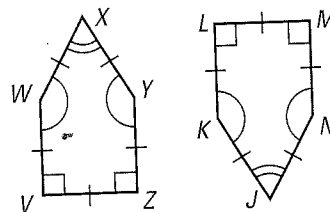
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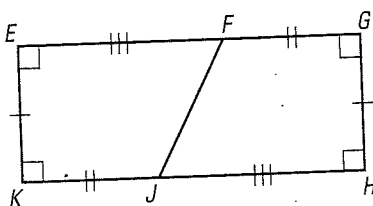
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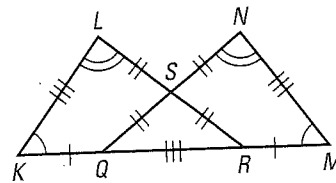
19.



20.

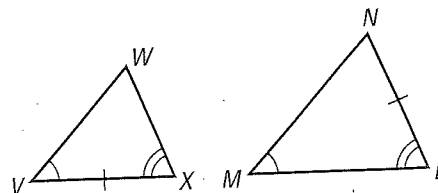


21.



22. **IDENTIFYING CORRESPONDING PARTS** Use the triangles shown in Exercise 17 above. Identify all pairs of congruent corresponding angles and corresponding sides.

23. **CRITICAL THINKING** Use the triangles shown at the right. How many pairs of angles are congruent? Are the triangles congruent? Explain your reasoning.



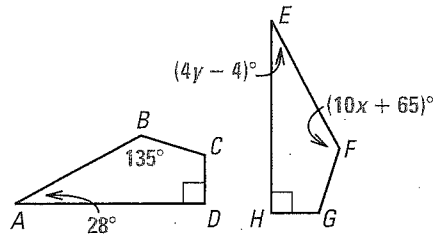
STUDENT HELP

HOMEWORK HELP

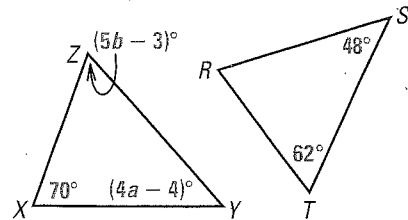
- Example 1: Exs. 10–22
 Example 2: Exs. 14, 24, 25
 Example 3: Exs. 26–29
 Example 4: Exs. 16–21, 23
 Example 5: Ex. 38

24. USING ALGEBRA Use the given information to find the indicated values.

24. Given $ABCD \cong EFGH$, find the values of x and y .

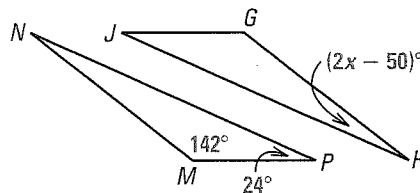


25. Given $\triangle XYZ \cong \triangle RST$, find the values of a and b .

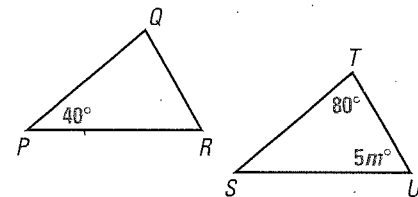


26. USING ALGEBRA Use the given information to find the indicated value.

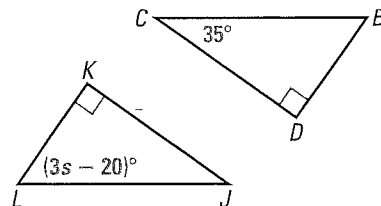
26. Given $\angle M \cong \angle G$ and $\angle N \cong \angle H$, find the value of x .



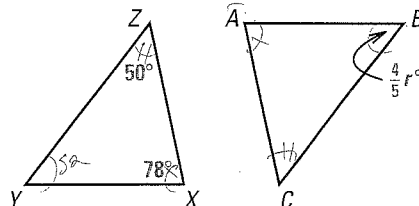
27. Given $\angle P \cong \angle S$ and $\angle Q \cong \angle T$, find the value of m .



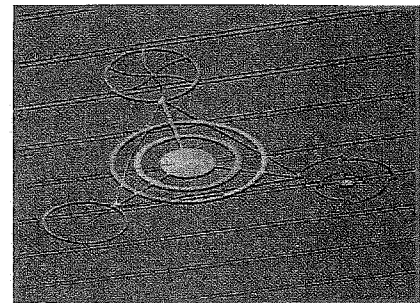
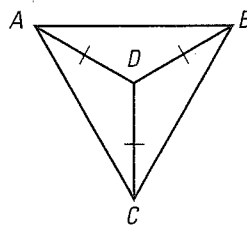
28. Given $\angle K \cong \angle D$ and $\angle J \cong \angle C$, find the value of s .



29. Given $\angle A \cong \angle X$ and $\angle C \cong \angle Z$, find the value of r .



CROP CIRCLES Use the diagram based on the photo. The small triangles, $\triangle ADB$, $\triangle CDA$, and $\triangle CDB$, are congruent.



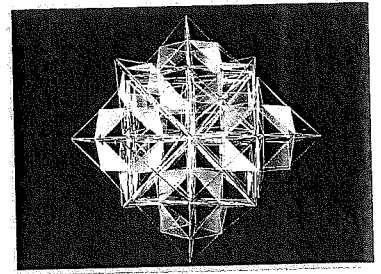
This pattern was made by mowing a field in England.

30. Explain why $\triangle ABC$ is equilateral.
 31. The sum of the measures of $\angle ADB$, $\angle CDA$, and $\angle CDB$ is 360° . Find $m\angle BDC$.
 32. Each of the small isosceles triangles has two congruent acute angles. Find $m\angle DBC$ and $m\angle DCB$.
 33. **LOGICAL REASONING** Explain why $\triangle ABC$ is equiangular.

FOCUS ON PEOPLE



HARRIET BRISSON is an artist who has created many works of art that rely on or express mathematical principles. The pattern used to arrange the triangles in her sculpture shown at the right can be extended indefinitely.

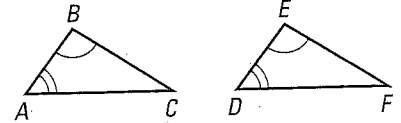


34. **SCULPTURE** The sculpture shown in the photo is made of congruent triangles cut from transparent plastic. Suppose you use one triangle as a pattern to cut all the other triangles. Which property guarantees that all the triangles are congruent to each other?

35. **DEVELOPING PROOF** Complete the proof of the Third Angles Theorem.

GIVEN $\angle A \cong \angle D, \angle B \cong \angle E$

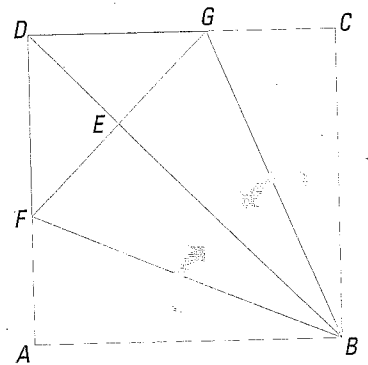
PROVE $\angle C \cong \angle F$



Statements	Reasons
1. $\angle A \cong \angle D, \angle B \cong \angle E$	1. ?
2. $m\angle ? = m\angle ?, m\angle ? = m\angle ?$	2. ?
3. $m\angle A + m\angle B + m\angle C = 180^\circ,$ $m\angle D + m\angle E + m\angle F = 180^\circ$	3. ?
4. $m\angle A + m\angle B + m\angle C =$ $m\angle D + m\angle E + m\angle F$	4. ?
5. $m\angle D + m\angle E + m\angle C =$ $m\angle D + m\angle E + m\angle F$	5. ?
6. $m\angle C = m\angle F$	6. ?
7. ?	7. Def. of $\cong \triangle$.

ORIGAMI Origami is the art of folding paper into interesting shapes. Follow the directions below to create a kite. Use your kite in Exercises 36–38.

- 1 Fold a square piece of paper in half diagonally to create \overline{DB} .
- 2 Next fold the paper so that side \overline{AB} lies directly on \overline{DB} .
- 3 Then fold the paper so that side \overline{CB} lies directly on \overline{DB} .



36. Is \overline{EB} congruent to \overline{AB} ? Is \overline{EF} congruent to \overline{AF} ? Explain.

37. **LOGICAL REASONING** From folding, you know that \overline{BF} bisects $\angle EBA$ and \overline{FB} bisects $\angle AFE$. Given these facts and your answers to Exercise 36, which triangles can you conclude are congruent? Explain.

38. **PROOF** Write a proof.

GIVEN $\overline{DB} \perp \overline{FG}, E$ is the midpoint of $\overline{FG}, \overline{BF} \cong \overline{BG},$
and \overline{BD} bisects $\angle GBF$.

PROVE $\triangle FEB \cong \triangle GEB$